Title: Forecast -- Trig Waves

Brief Overview:

> Students use the Internet to access average high/low temperatures for various cities. Students enter the monthly temperature data into a TI-82 or TI-83 calculator and produce a statistical plot of the data. They see that the best fit for the data is a cosine graph. Students determine the particular cosine equation that best fits the statistical data. Reversing the process, students interpret cosine graphs and match them to cities.

Links to Standards:

- **Mathematics as Problem Solving**
  Students will analyze data to model real-world climate statistics and make conclusions based on comparisons.

- **Mathematics as Communications**
  Students will interactively discuss translations of cosine curves and follow up with a written summary.

- **Mathematics as Reasoning**
  Students will make conjectures based on collected Internet data and graphs generated from the data.

- **Mathematical Connections**
  Science - Students will see that temperature data can be modeled by a trigonometric function.
  Geography - Students will find cities on an Internet map and make generalizations concerning location and its effect on temperature patterns.

- **Trigonometry**
  Students will analyze graphs to write an equation of a cosine curve in the form $y = A\cos(Bx) + D$. Students will relate the parameters A, B, and D to the Amplitude, Period, and Vertical Shift of the cosine graph and will adjust the values of these parameters to obtain the best fit to the data.

- **Statistics**
  Students will enter temperature data into a TI-82/83 calculator and use the STAT PLOT function to obtain a graph of the data.

Grade/Level:

> Grades 10-12, Trigonometry and Pre-Calculus

Duration/Length:

This activity will take approximately two periods on a ninety-minute block.

Prerequisite Knowledge:

Students should have the following skills:

- Simple Internet navigation techniques
- Basic use of the TI-82/83   (Statistics input, Stat Plot setup, and graphing)
- Basic knowledge of the cosine function and its graph. (Period, Amplitude, Vertical Shift)

Objectives:

Students will:

- work cooperatively in groups.
- access the Internet for monthly high and low temperatures of specific cities.
- use a TI-82/83 for statistics input and graphing.
- interpret results and draw conclusions.
- express answers clearly in writing.
- use trigonometry skills to write an equation from a graph.
- use a graphing calculator to verify analytical results

Materials/Resources/Printed Materials:

- Graphing Calculator (TI-82/83)
- Computer with Internet access
- Worksheets: Student and Teacher Resources

Development/Procedures:

- Preview Internet access procedures.
- Divide students into groups of 2-5.
- Follow self-directed worksheets.

Performance Assessment:

Assessment will be based on observation of ability to access and use Internet data as well as evaluation of completed worksheets.

Extension/Follow Up:

- Access other Internet sites for weather, such as http://www.worldclimate.com (so that students will become aware that there are many sites that give weather information, some of which have formats which are more complete or easier to interpret).
- Rewrite equations in the form $y = A\sin B(x-C) + D$.
- Compare temperature graphs for cities with the same latitude and describe how other factors such as proximity to the ocean or elevation affect yearly temperatures.

Authors:

Vicky A. Andrews	Debbie DeMaria	Francis W. Shelton
West Springfield H. S.	Lake Braddock Secondary	Pulaski County High School
Fairfax County, VA	Fairfax County, VA	Pulaski County, VA

ACCESSING   THE   INTERNET

Log on.

Double click on Netscape Navigator.

In the location box type:

> **http://www.nbc4.com**

Click on Weathernet 4.

Click on Intellicast.

A. Click on "USA".

B.   Locate desired city on map and click.

   I. Click on "city almanac" in the left column.
   II. Copy high temperatures onto given chart below.

C. Repeat steps A and B above until chart is complete.

## MONTHLY AVERAGE HIGH TEMPERATURES (° F)

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
Miami												
Chicago												
Dallas												
Fairbanks												

GRAPHING THE DATA

INSTRUCTIONS:

1 a. Press **STAT.**
  b. Choose **EDIT.**
  c. In **L1** enter 0, 1, 2 ....., 11 to represent the months January through December.
  d. In **L2** enter Miami temperatures.

2. a. Press **Stat Plot** (2nd Y=).
  b. Choose **1.**
  c. Choose **On.**
  d. Choose Type: **Scatter Plot**.
  e. X list: L1; Y list: L2
  f. Press **Window**: [-1,12]; Xscale 1; [0,100]; Yscale 10.
  g. Press **Graph.**

3. a. Copy graph on axes provided.
  b. Is this temperature data periodic? _____
  c. What type of graph does this appear to be?_____
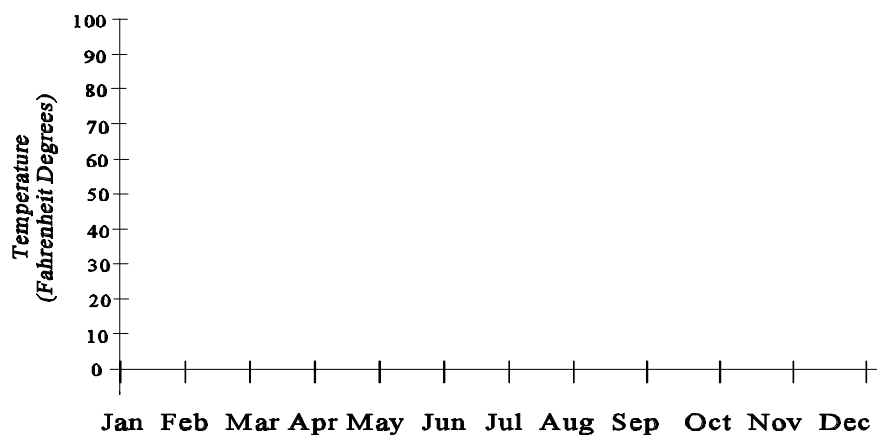

MIAMI
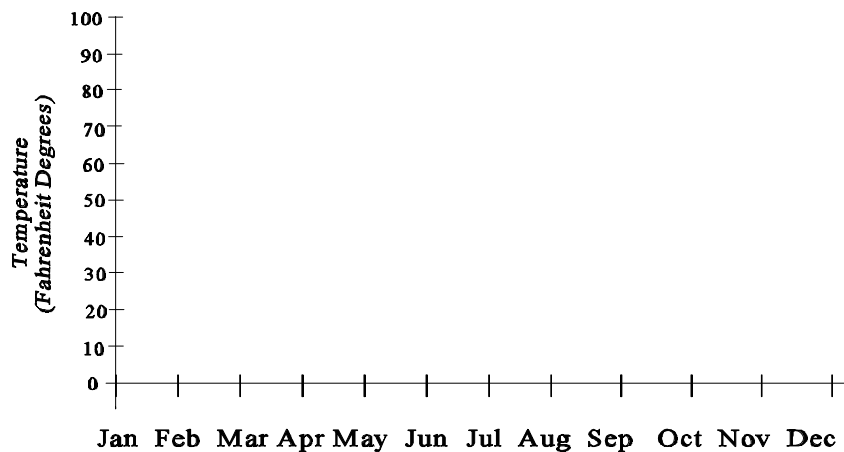
Temperature (Fahrenheit Degrees)

Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec

4. a. What is the amplitude?_____
  b. What is the period?_____
  c. Write the equation of the curve above in the form y = A cos (Bx) + D.

  _____

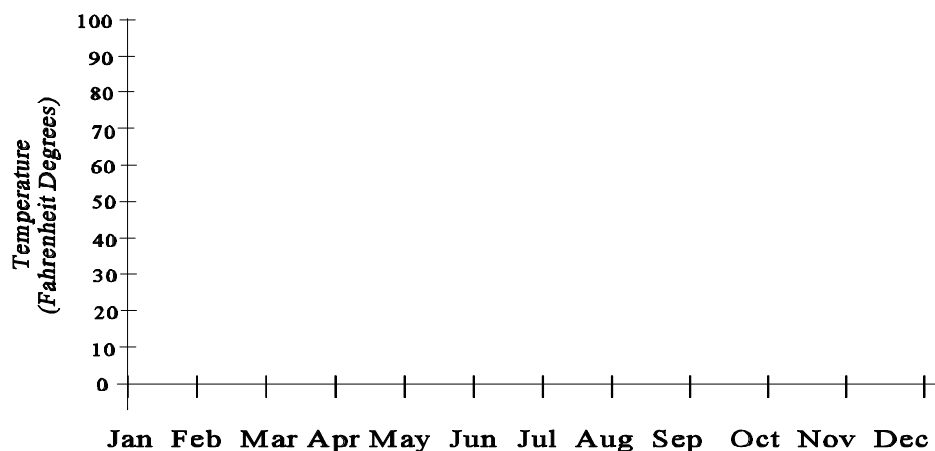 d. Verify your answer by entering your equation in Y1 and graphing on top of the stat plot.

Repeat Instructions 1-3 on the previous page for Chicago, Dallas, and Fairbanks.

CHICAGO

a. What is the amplitude?_____

b. What is the period?_____

c. Write the equation of the curve above in the form $y = A \cos (Bx) + D$.
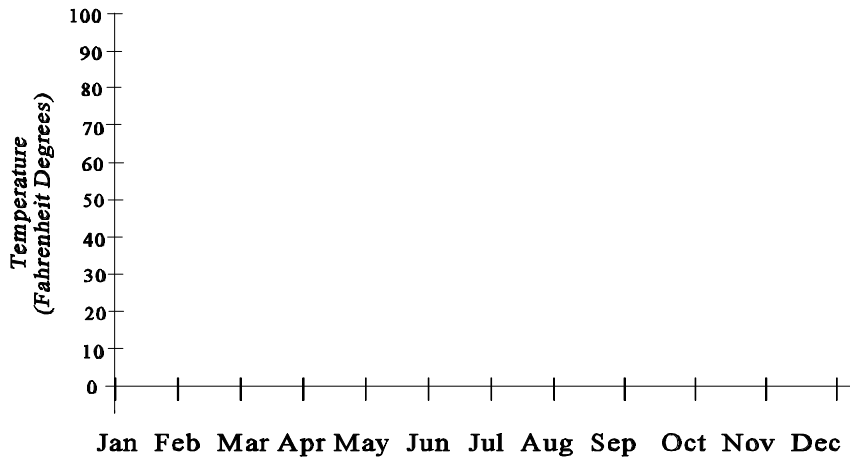
   _____

d. Verify your answer by entering your equation in Y1 and graphing on top of the stat plot.

DALLAS

a. What is the amplitude?_____
b. What is the period?_____
c. Write the equation of the curve above in the form $y = A \cos (Bx) + D$.

   _____

d. Verify your answer by entering your equation in Y1 and graphing on top of the stat plot.

FAIRBANKS

Temperature
(Fahrenheit Degrees)

100
90
80
70
60
50
40
30
20
10
0

Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec

    a.   What is the amplitude?_____

    b.   What is the period?_____

    c.   Write the equation of the curve above in the form $y = A \cos (Bx) + D$.

        _____

    d. Verify your answer by entering your equation in Y1 and graphing on top of the stat plot.

SUMMARY

1. What is the same about all the **A** values?_____
    Explain why?_____

2. Where in the world do you think the A-values would have the opposite sign?

    Give 2 specific cities _____   _____

    Why did you choose these cities? _____

3. What is true about the periods of the graphs of all cities observed?_____

    Why?_____
4. Which city shows the greatest variation in average high temperatures?_____
5. List the four cities in order from greatest to least variation.

      _____

      _____

      _____

      _____

TEMPERATURE TRANSFORMATIONS

PART **A**

1. Log on and access Internet.

2. In the location box, type:

    http://www.usatoday.com/weather/waverage.htm

3. Scroll down to University of Utah WWW links and choose **"Normal high temperatures."**

4. Locate **Washington National Airport, D.C.** and record the data in the chart below.

5. Press **Back**. Choose "Normal low temperatures" and complete the table for Washington D.C.

Wash.D.C.	J A N	F E B	M A R	A P R	M A Y	J U N	J U L	A U G	S E P	O C T	N O V	D E C
HIGH TEMP												
LOW TEMP												

6. a. Enter the data on your TI 82/83. Use L1 for months (0-11), L2 for Highs, and L3 for Lows

    b. Use L1 and L2 to create Stat Plot #1 for Highs and L1 and L3 to create Stat Plot #2 for Lows.
    (For specific instructions, please see Activity 1.)

    c. Graph both Stat Plots on the same axes and sketch the result below:

Washington, D.C.
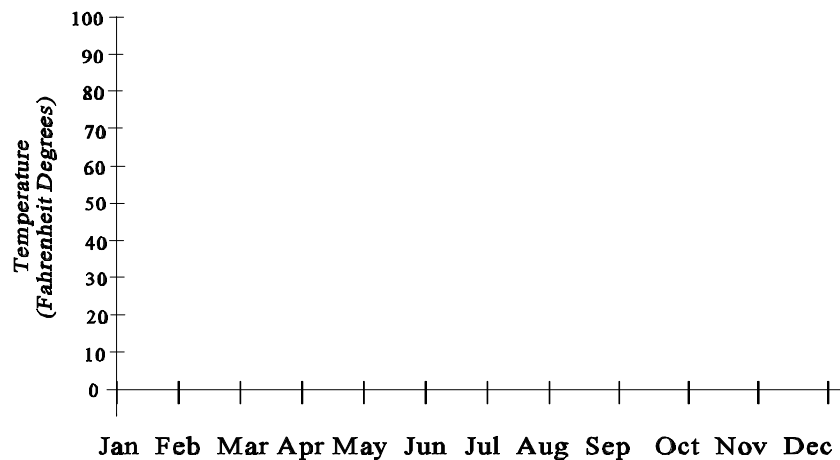
7. Write an equation, h(x), for the function graphed in Stat Plot #1 (average high temperatures in Wash. D.C.).

    h(x) =_____

8. a. Describe the graph of Stat Plot #2 (average low temperatures) in terms of the highs.

    _____

    b. Write an equation, w(x), to represent the low temperatures in terms of the function h(x).

    _____

    c. What type of transformation is represented by these two graphs?

    _____

    d. Check to see if this is the correct transformation by entering w(x) in **Y1=** . Graph and compare results to Stat Plot #2.   Discuss with your partner(s). What do you conclude?

    _____

PART B

1. In the location box, type:

| gopher://gilgamesh.ho.BoM.GOV.AU/ |

2. Click on **Australian Climate Data**.
3. Click on **Mean Temperatures and Rainfall for Australian Locations**.
4. Click on **Capital Cities**.
5. Scroll to **Melbourne** and record data in the chart below. (Note that temperatures are degrees Celsius.)

Melbourne (° C)	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
HIGH TEMP												

6. Enter the data in Celsius in L4 of your TI 82/83.

7. To compare the data to previously collected data, you will need to convert to **Fahrenheit**. (Discuss with your group how to do this.) When you have the conversion equation, enter the data in L5 of your TI 82/83.   Note: To check your results, return to the "Intellicast" WWW weather site accessed in Activity 1 and select "**Dr. Dewpoint**" and **"Questions from Visitors**." (Or in header of L5 column type (9/5)(L4) + 32 and press enter --this changes all Melbourne's °C to °F.)

Record the **Fahrenheit** temperatures below.

Melbourne (°F)	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
HIGH TEMP												

8. Create a Stat Plot using L1 and L5 (degrees F for Melbourne)   Copy the graph here:

Melbourne, Australia
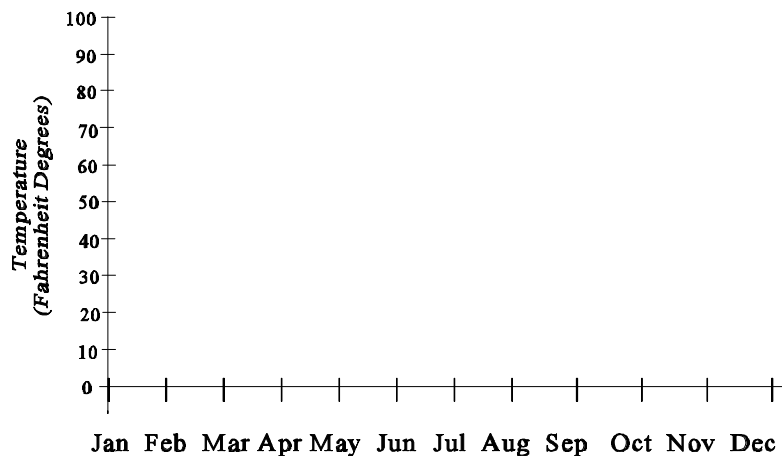
9. What do you observe about this graph? How is it the same as other graphs in this lesson?

_____

How is it different from other graphs?   _____

How do you account for this difference geographically?   _____

10. a. Write the equation for this graph in the form **y = Acos (Bx) + D**.

_____

   b. What is the amplitude?_____   the period? _____

   c. Verify your answer using your calculator.

   d. Equation (a) could be rewritten in the form y = AcosB(x - C) + D to obtain

   $$y = -10.5 \cos((\pi/6)(x - 6)) + 68$$

   What does the "C" value 6 represent in terms of the real world?_____

   _____

11.   Review all the graphs you have made in Activities 1 and 2.

   Find one city whose graph approximates a reflection of Melbourne's graph._____

   State the equation of the line of reflection.   _____

12. Comparing Melbourne's graph with the other graphs, what kind of mathematical transformation(s) would best describe the real-world data? (Vertical shift, Horizontal or Phase shift, Reflection, Dilation)
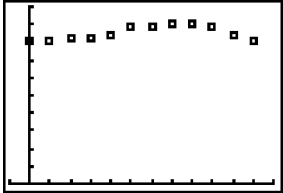
   Write complete sentences to justify your answer.
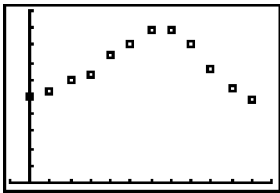
_____

_____

_____

TRIG WAVES ASSESSMENT

The following graphs represent average maximum temperatures in four world cities.
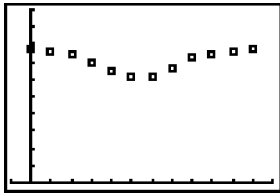(Xscale = 1, Yscale = 10) Note: 0 on the Xscale is January
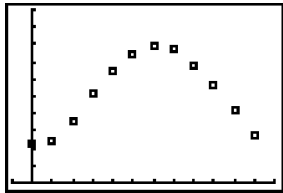
City A

City B

City C

City D

1. Match each of the graphs with the appropriate city in the table below:

City	Location	Graph
Honolulu	21° N 157°E	
Johannesburg	26.13°S 28.23°E	
Madrid	40.47°N 3.45° W	
Montreal	45.47°N 73.75° W	

2. Choose 2 of the four cities above. Write the equation of each in the form y = Acos(Bx) + D.
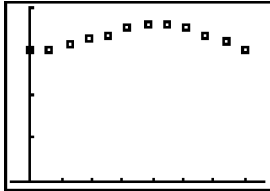
City	
	Equation

3. What is the role of latitude in determining the nature of the average maximum temperature
    graph of a city?

TEACHER RESOURCE

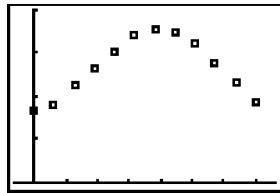## MONTHLY AVERAGE HIGH TEMPERATURES (°F)

	JAN		MAR	APR		JUN	JUL		SEP	OCT		DEC
Miami		75	79		84	88		90	88		84	75
	28	34		59	70		82	82		64	48	
Dallas	55		66	75		91	97		90	79		59
Fairbanks		7	25		59	70		66	55		10	1

4. a. What is the amplitude?_____

   b. What is the period?__12___

$$y = -7.5 \cos(\pi$$
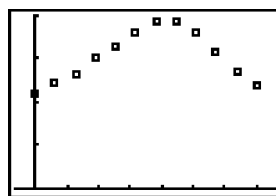
Activity 1, Page 3

CHICAGO

   __28__

b. What is the period?_____

c. Write the equation of the curve above in the form y = A cos (Bx) + D.
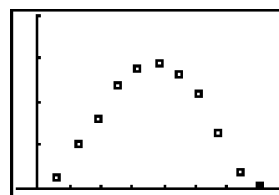
   y = -28 cos (x/6) + 56

DALLAS

a. What is the amplitude?__21__
b. What is the period?__12__
c. Write the equation of the curve above in the form $y = A \cos(Bx) + D$.

$y = -21 \cos(\pi x/6) + 76$

Activity 1, Page 4

FAIRBANKS

a. What is the amplitude?__37__
b. What is the period?__12__
c. Write the equation of the curve above in the form $y = A \cos(Bx) + D$.
$y = -37 \cos(\pi x/6) + 36$

SUMMARY

1. What is the same about all the **A** values?__They are all negative.__
   Explain why?__They are located in the northern hemisphere.__

2. Where in the world do you think the A-values would have the opposite sign?
   Give 2 specific cities__Answers will vary ex: Santiago, Chile; Sydney, Australia__
   Why did you choose these cities?__They are located in the southern hemisphere.__

3. What is true about the periods of the graphs of all cities observed?__All the periods are 12.__
   Why?__There are 12 months in a year.__

4. Which city shows the greatest variation in average high temperatures?__Fairbanks, Alaska__
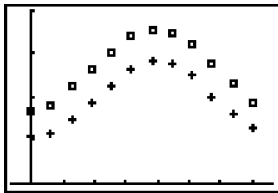
5. List the four cities in order from greatest to least variation.
    Fairbanks
    Chicago
    Dallas
    Miami

Activity 2 Page 1

Wash.D.C	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
HIGH TEMP	42.3	45.9	56.5	66.7	76.2	84.7	88.5	86.9	80.1	69.1	58.3	47
LOW TEMP	26.8	29.1	37.7	46.4	56.6	66.5	71.4	70	62.5	50.3	41.1	31.7

6.

Activity 2, Page 2

7.   WASHINGTON D.C. Write an equation, h(x), for the function graphed in Stat Plot #1 (average high temperatures in Washington D.C.)

$$h(x) = \underline{\;-23.1\cos(\pi x/6) + 65.4\;}$$

8. a. Describe the graph of Stat Plot #2 (average low temperatures) in terms of the highs.

    All values are below the corresponding high temperatures.

    b. Write an equation, w(x), to represent the low temperatures in terms of the function h(x).

    $$w(x) = \underline{\;h(x) - 16\;}$$

    c. What type of transformation is represented by these two graphs?

    Vertical translation 16 units

    d. Check to see if this is the correct transformation by entering w(x) in **Y1=** . Graph and compare results to Stat Plot #2. Discuss with your partner(s). What do you conclude?

    Answers will vary.

Melbourne (°C)	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
HIGH TEMP	25.8	25.7	23.7	20.2	16.6	13.9	13.3	14.8	17.1	19.5	21.8	24.1

Activity 2, Page 3

Note: To have the calculator convert the Celsius temperatures to Fahrenheit, in the header of L5 type (9/5)L4+32 and press enter. This changes all of Melbourne's °C to °F.

Melbourne (°F)	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
HIGH TEMP	78.44	78.26	74.66	68.36	61.88	57.02	55.94	58.64	62.78	67.1	71.24	75.38
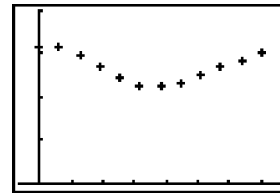
MELBOURNE

9. What do you observe about this graph? How is it the   same as other graphs in this lesson?
        The graph starts up. Period is the same. Cosine curve.
    How is it different from other graphs?   The A value is positive.
    How do you account for this difference geographically?   Southern hemisphere
                                             Activity 2, Page 4
10. a. Write the equation for this graph in the form **y = Acos (Bx) + D**.
                    y = 10.5 cos (πx/6) + 68

    b. What is the amplitude? 10.5 the period? 12
    d. What does the "C" value 6 represent in terms of the real world?
        The highest point of the curve occurs 6 months later (or earlier) than in the northern hemisphere.

11.   Find one city whose graph **approximates** a reflection of Melbourne's graph.   Miami
       State the equation of the line of reflection.   y = 75

12. Comparing Melbourne's graph with the other graphs, what kind of mathematical transformation(s) would best describe the real-world data? (Vertical shift, Horizontal or Phase shift, Reflection, Dilation)   Answers will vary.

TRIG WAVES ASSESSMENT

1. Match each of the graphs with the appropriate city in the table below:

City	Location	Graph
Honolulu	21° N 157°E	B
Johannesburg	26.13°S 28.23°E	D
Madrid	40.47°N 3.45° W	C
Montreal	45.47°N 73.75° W	A

2. Choose 2 of the four cities above. Write the equation of each in the form $y = A\cos(Bx) + D$.
    Note: All answers are approximate.

City	Answers computed from data	Estimated answers from graphs
HONOLULU	$y = -4.5 \cos(\pi x/6) + 85.5$	$y = -5 \cos(\pi x/6) + 85$
JOHANNESBURG	$y = 8.2 \cos(\pi x/6) + 70$	$y = 10 \cos(\pi x/6) + 70$
MADRID	$y = -20 \cos(\pi x/6) + 69$	$y = -20 \cos(\pi x/6) + 70$
MONTREAL	$y = -28.6 \cos(\pi x/6) + 51$	$y = -30 \cos(\pi x/6) + 50$

3. What is the role of latitude in determining the nature of the average maximum temperature
    graph of a city?

    The closer to the equator the smaller the amplitude; the farther from the equator the greater the
    amplitude. In the northern hemisphere, the A-value is negative; in the southern hemisphere,
    positive.